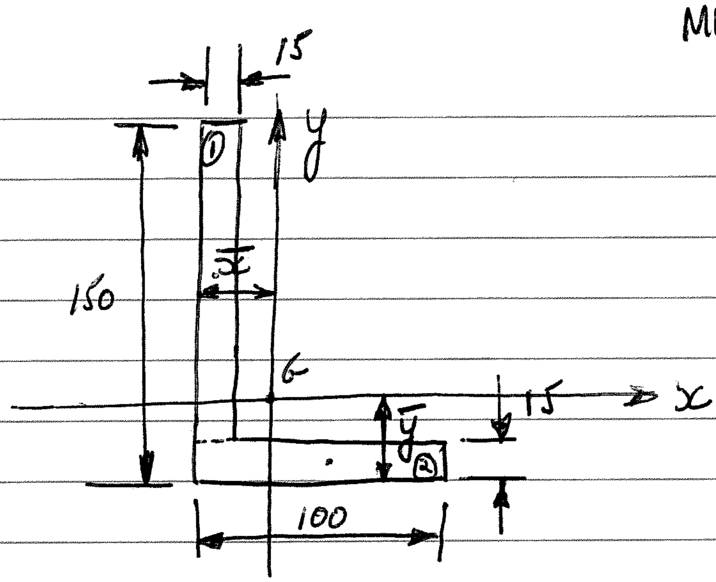
Q1



Centroid of Area

$A_1 = 135 \times 15 = 2025$

$A_2 = 100 \times 15 = 1500$

$A_{TOT} = \quad = 3525$

$3525 . \bar{x} = 2025 . 7.5 + 1500 . 50$

$\bar{x} = \underline{25.59}$

$3525 . \bar{y} = 2025 . 82.5 + 1500 . 7.5$

$\bar{y} = \underline{50.59}$ [10 marks]

Principal 2nd Moments of Area

$$I_x = \frac{15 . 135^3}{12} + 2025 . 31.91^2 + \frac{100 . 15^3}{12} + 1500 . 43.09^2$$

$$= 3,075,469 + 2,061,952 + 28,125 + 2,785,122$$

$$= \underline{7,950,668} \text{ mm}^4$$

$$I_y = \frac{135 . 15^3}{12} + 2025 . 18.09^2 + \frac{15 . 100^3}{12} + 1500 . 24.41^2$$

$$= 37,969 + 662,677 + 1,250,000 + 893,772$$

Q1 (cont)

$$\therefore I_y = \underline{2,844,418 \text{ mm}^4}$$

$$I_{xy} = 0 + 2025 \cdot 31.91 \cdot -18.09 + 1500 \cdot -43.09 \cdot 24.41$$

$$= -1,168,935 - 1,577,740$$

$$= \underline{-2,746,675 \text{ mm}^4}$$

$$\text{Circle Centre } C = \frac{I_x + I_y}{2} = \underline{5,397,543}$$

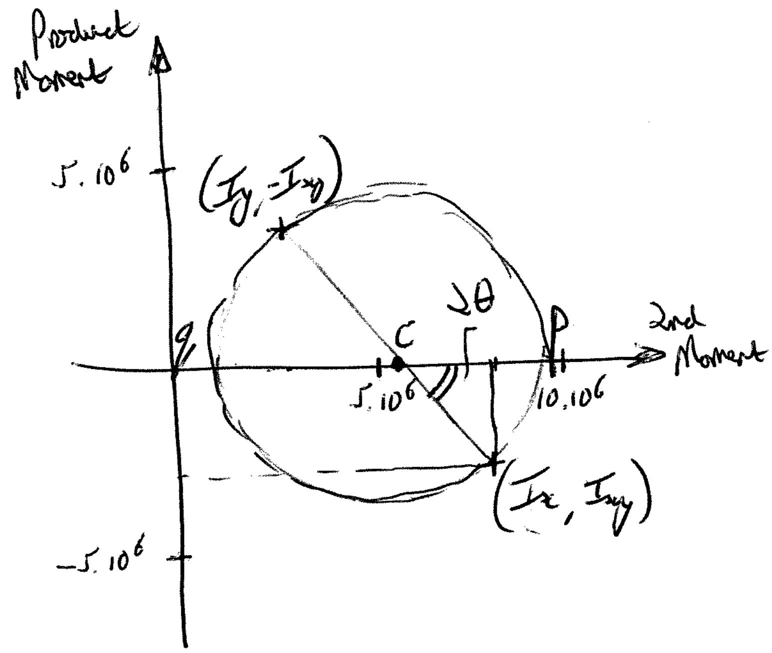
$$\text{Radius } R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = \underline{3,750,023}$$

$$I_p = C + R = \underline{9,147,566 \text{ mm}^4}$$

$$I_q = C - R = \underline{1,647,520 \text{ mm}^4}$$

[16 marks]

Angle

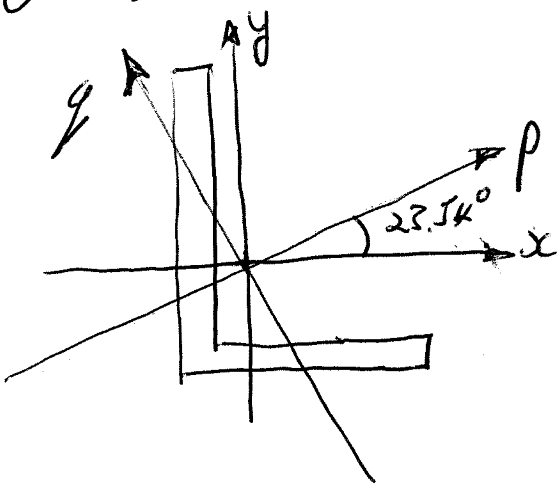


$$\sin 2\theta = \frac{I_{xy}}{R}$$

$$= \frac{2,746,675}{3,750,023}$$

$$2\theta = 47.09^\circ$$

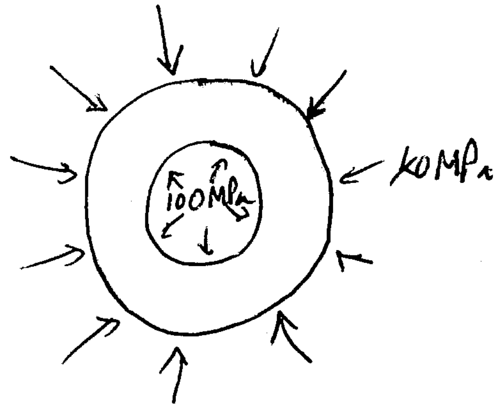
$$\therefore \theta = \underline{23.54^\circ}$$



p axis is 23.54° anticlockwise from the x-axis

[7 marks]

Q2



Lamé's eqns:

$$\sigma_r = A_1 - \frac{B_1}{r^2}$$

$$\sigma_\theta = A_1 + \frac{B_1}{r^2}$$

Using radial equation:

$$-100 = A_1 - \frac{B_1}{20^2} \quad ①$$

$$-40 = A_1 - \frac{B_1}{40^2} \quad ②$$

Subtracting ① from ②

$$-40 + 100 = -\frac{B_1}{40^2} + \frac{B_1}{20^2}$$

$$B_1 = \frac{60}{-\frac{1}{40^2} + \frac{1}{20^2}}$$

$$= \underline{\underline{32,000}}$$

From ① $$A_1 = -100 + \frac{32,000}{20^2}$$

$$= \underline{\underline{-20}}$$

At the inner wall $$\sigma_{\theta_{inner}} = A_1 + \frac{B_1}{20^2} = -20 + \frac{32,000}{20^2} = \underline{\underline{60\text{ MPa}}}$$

Q2 (cont)

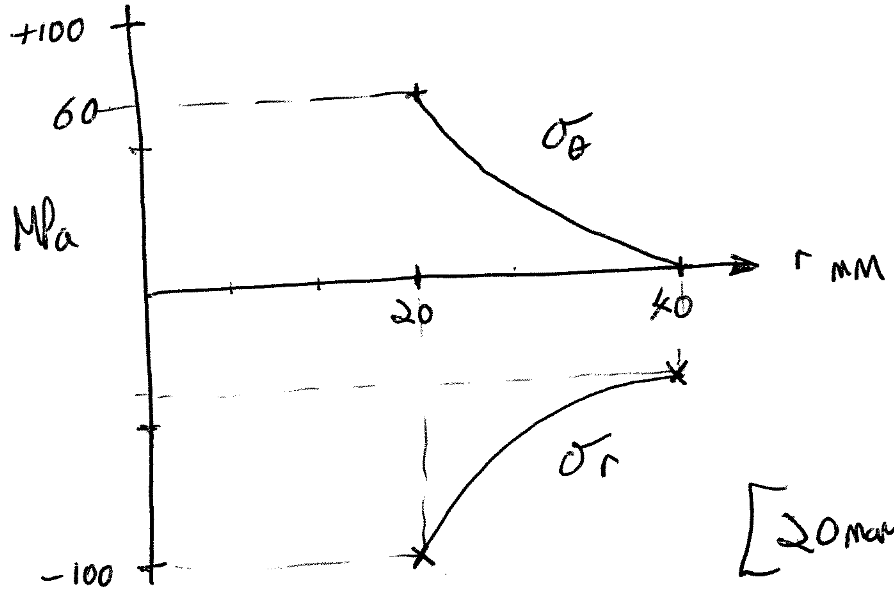
At the outer wall

$$\sigma_{\theta_{outer}} = A_1 + \frac{B_1}{40^2}$$

$$= -20 + \frac{32000}{40^2}$$

$$= \underline{\underline{0}}$$

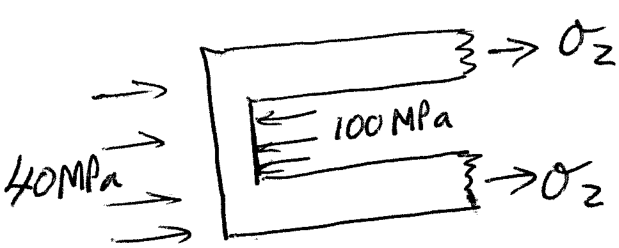
<u>Distribution</u>



[20 marks]

<u>Axial Stress</u>

Free Body Diagram



<u>Equilibrium</u>

$$100 \cdot 10^6 \cdot \cancel{\pi}(0.02)^2$$

$$= 40 \cdot 10^6 \cancel{\pi}(0.04)^2 + \sigma_z \cancel{\pi}(0.04^2 - 0.02^2)$$

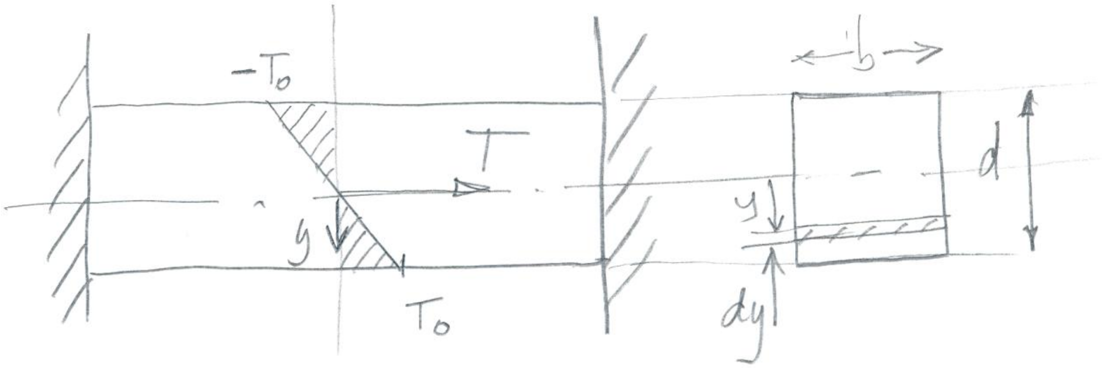
$$4 \cdot 10^4 = 6.4 \cdot 10^4 + \sigma_z (1.2 \cdot 10^3)$$

$$\sigma_z = \underline{\underline{-20 \text{ MPa}}}$$

compressive

[13 marks]

Q3



$T = T_o \text{ x} \dfrac{2y}{d}$

<u>Axial Force Equilibrium</u> (Equation (2))

$$P = E\bar{\varepsilon}A - E\alpha\int_A T dA$$

$$\int_A T dA = \frac{2T_o b}{d}\int_{\frac{-d}{2}}^{\frac{d}{2}} y\ dy = \frac{2T_o b}{d}\left[\frac{y^2}{2}\right]_{\frac{-d}{2}}^{\frac{d}{2}} = 0$$

Also,
$\bar{\varepsilon} = 0$ $\quad\therefore$ <u>P = 0</u>

<u>Moment Equilibrium</u> (Equation (3))

$$M = \frac{EI}{R} - E\alpha\int_A T\ ydA$$

$$\int_A TydA = \frac{2T_o b}{d}\int_{-\frac{d}{2}}^{\frac{d}{2}} y^2 dA = \frac{2Tob}{d}\left[\frac{y^3}{3}\right]_{\frac{-d}{2}}^{\frac{d}{2}}$$

$$= \frac{2T_o b}{d}\left[\left(\frac{d^3}{24}\right) - \left(\frac{-d^3}{24}\right)\right] = \frac{T_o bd^2}{6}$$

Also,

$$1/R = 0, \therefore \text{M} = \frac{-E\alpha bd^2 T_o}{6}$$

Using Equation (1) (with ε = (1/R = 0)

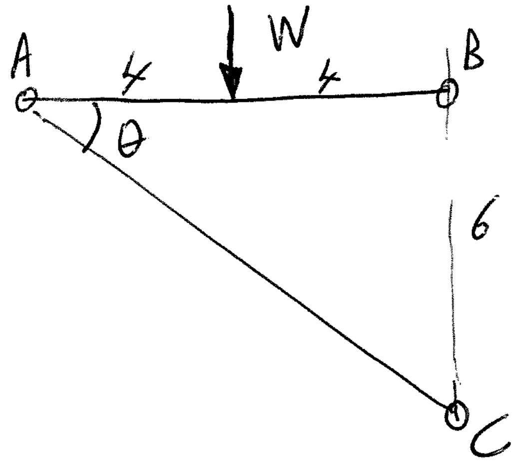
$$\sigma_x = \left(\bar{\varepsilon} + \frac{y}{R} - \alpha T \right) = -E\alpha.T$$

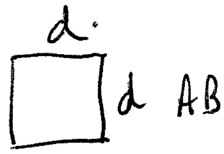
$$\therefore \sigma_x = -E\alpha \; T_o \frac{2y}{d}$$

ie

$$\sigma_X = \frac{-2E\alpha T_o}{d} \, y$$

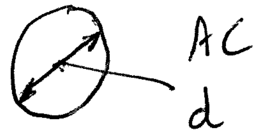
Q4



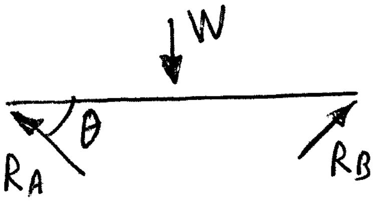


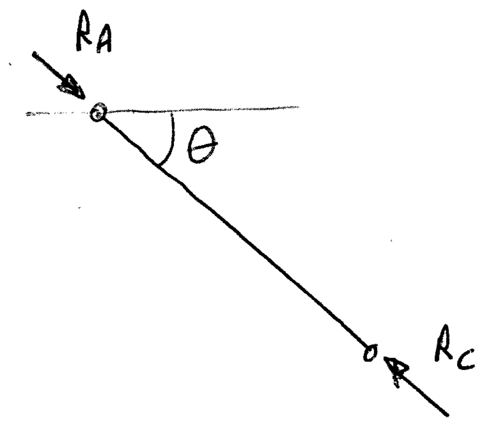
$\tan\theta = \frac{6}{8}$

$\theta = \underline{36.87^\circ}$



Free Body Diagrams





From symmetry $R_A = R_B$

Equilibrium $2 R_A \sin\theta = W$

$\therefore \quad R_A = \frac{W}{2\sin 36.87} = \underline{\underline{\frac{W}{1.2}}}$

2 Force member

$\therefore$ R_A is along line of AC & compression

Equilibrium $R_A = R_C$

Bending in AB

$M_{max} = \frac{W}{2} \times 4\text{m} = \frac{W L_{AB}}{4}$

$= \underline{\underline{2W}}$ Nm

$\sigma_{max} = \frac{M^{max} y}{I} = \frac{\frac{\cancel{2W}\ W L_{AB}}{4} \cdot \frac{d}{2}}{\frac{d \cdot d^3}{12}} = \cancel{\frac{12W}{d^3}} \quad \underline{\underline{\frac{3 W L_{AB}}{2 d^3}}}$

Q (cont)

4 Buckling Load of Strut

$$P_{crit} = \frac{\pi^2 E I}{L_{AC}^2}$$

$$I = \frac{\pi d^4}{64}$$

$$P_{crit} = \frac{\pi^2 E \pi d^4}{64 L_{AC}^2}$$

$$= \frac{\pi^3 E d^4}{64 L_{AC}^2}$$

Collapse in the Structure

Yield in AB when $W_{crit} = \frac{\sigma_y . 2d^3}{3 L_{AB}}$

$$= \frac{300.10^6 . 2.(0.04)^3}{3 . 8}$$

$$= 1600 \text{ N}$$

Collapse in AC

when $P_{crit} = \frac{\pi^3 . 210.10^9 .(0.03)^4}{64 . 10^2}$

$$= 824 \text{ N}$$

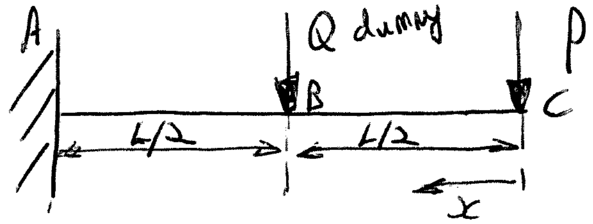
$L_{AC} = \sqrt{8^2 + 6^2} = 10 \text{m}$

$\therefore W_{crit} = 1.2 \times 824$

$= 989 \text{ N}$ i.e. Max W

Mechanism of collapse is buckling in strut AC. [33 marks]

Q5



<u>Between C & B</u>

$$M = -Px$$

$$U_{CB} = \int_0^{L/2} \frac{M^2 dx}{2EI} = \int_0^{L/2} \frac{P^2 x^2}{2EI} dx$$

$$= \frac{P^2}{2EI}\left[\frac{x^3}{3}\right]_0^{L/2}$$

$$= \underline{\underline{\frac{P^2 L^3}{48EI}}}$$

<u>Between B & A</u>

$$M = -Px - Q\left(x - \tfrac{L}{2}\right)$$

$$U_{BA} = \int_{L/2}^{L} \frac{\left[Px + Q\left(x - \frac{L}{2}\right)\right]^2}{2EI} dx$$

$$U_{TOT} = U_{CB} + U_{BA} = \frac{P^2 L^3}{48EI} + \frac{1}{2EI}\int_{L/2}^{L} \left[Px + Q\left(x - \tfrac{L}{2}\right)\right]^2 dx$$

$$= \frac{P^2 L^3}{48EI} + \frac{1}{2EI}\int_{L/2}^{L} P^2x^2 + Q^2\left(x - \tfrac{L}{2}\right)^2 + 2PQx\left(x - \tfrac{L}{2}\right) dx$$

<u>Deflection at C</u>

$$\delta_C = \left(\frac{\partial U_{TOT}}{\partial P}\right)_{Q=0} = \frac{PL^3}{24EI} + \frac{1}{2EI}\int_{L/2}^{L} 2Px^2 dx$$

Q5 (cont)

$$\delta_C = \frac{PL^3}{24EI} + \frac{2P}{2EI}\left[\frac{x^3}{3}\right]_{L/2}^{L}$$

$$= \frac{PL^3}{24EI} + \frac{P}{EI}\left[\frac{L^3}{3} - \frac{L^3}{24}\right]$$

$$= \frac{PL^3}{EI}\left[\frac{1}{24} + \frac{1}{3} - \frac{1}{24}\right]$$

$$\delta_C = \frac{PL^3}{3EI}$$ ✓ i.e. standard cantilever formula.

[20 marks]

Deflection at B

$$\delta_B = \left(\frac{\partial U_{tot}}{\partial Q}\right)_{Q=0} = \frac{1}{2EI}\int_{L/2}^{L} 2Q\left(x-\frac{L}{2}\right)^2 + 2Px\left(x-\frac{L}{2}\right)\,dx$$

$$= \frac{2P}{2EI}\int_{L/2}^{L}\left(x^2 - \frac{xL}{2}\right)dx$$

$$= \frac{\cancel{2}P}{\cancel{2}EI}\left[\frac{x^3}{3} - \frac{x^2L}{4}\right]_{L/2}^{L}$$

$$= \frac{P}{EI}\left[\frac{L^3}{3} - \frac{L^3}{4} - \frac{L^3}{24} + \frac{L^3}{16}\right]$$

$$= \frac{PL^3}{EI}\left[\frac{16-12-2+3}{48}\right]$$

$$= \frac{5PL^3}{48EI}$$ ✓

[13 marks]